Analysis of Curved and Skewed Steel Girder Bridges during Construction

Western Bridge Engineers' Seminar

Session 8A Bridge Design Policies
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Domenic Coletti
Andres Sanchez
Brandon Chavel
HDR Engineering

Cagri Ozgur
Rizzo Associates

Donald White
Georgia Institute of Technology

ACKNOWLEDGEMENTS

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Guidelines for Analytical Methods and Construction Engineering of Curved and Skewed Steel Girder Bridges

NCHRP Report 725

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Presentation Outline

- Introduction
- Current 2D Models
- Improved 2D Models
- Case Studies
- Conclusions

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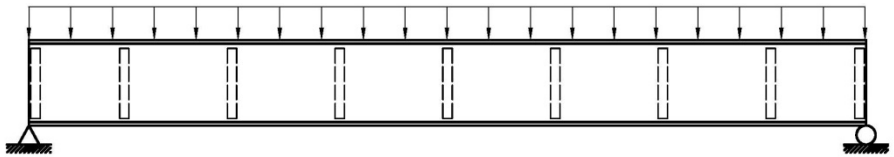
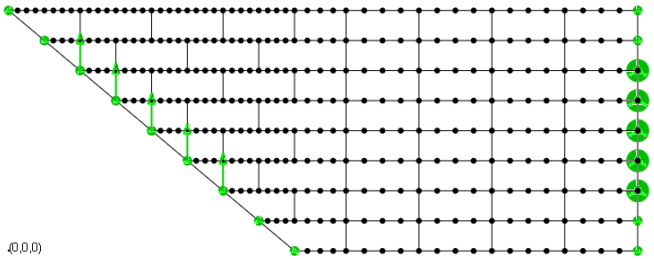
Introduction: Methods of Analysis

Prediction of major-axis bending stresses, f_b, flange lateral bending stresses, f_ℓ, vertical displacements, girder layovers, etc.

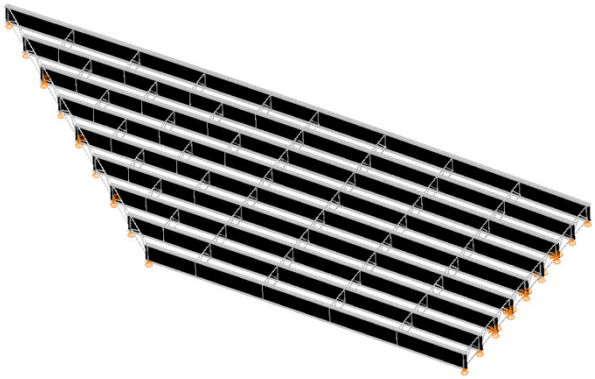
1D Line-Girder:



2D-Grid:



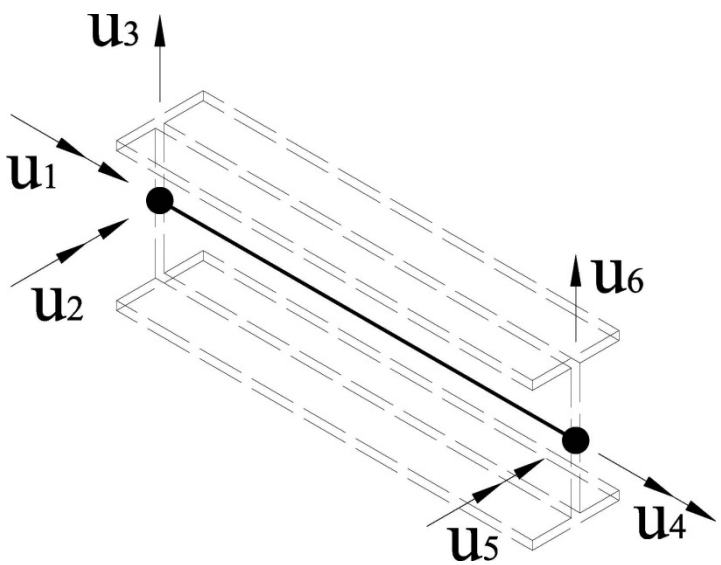
3D FEA:



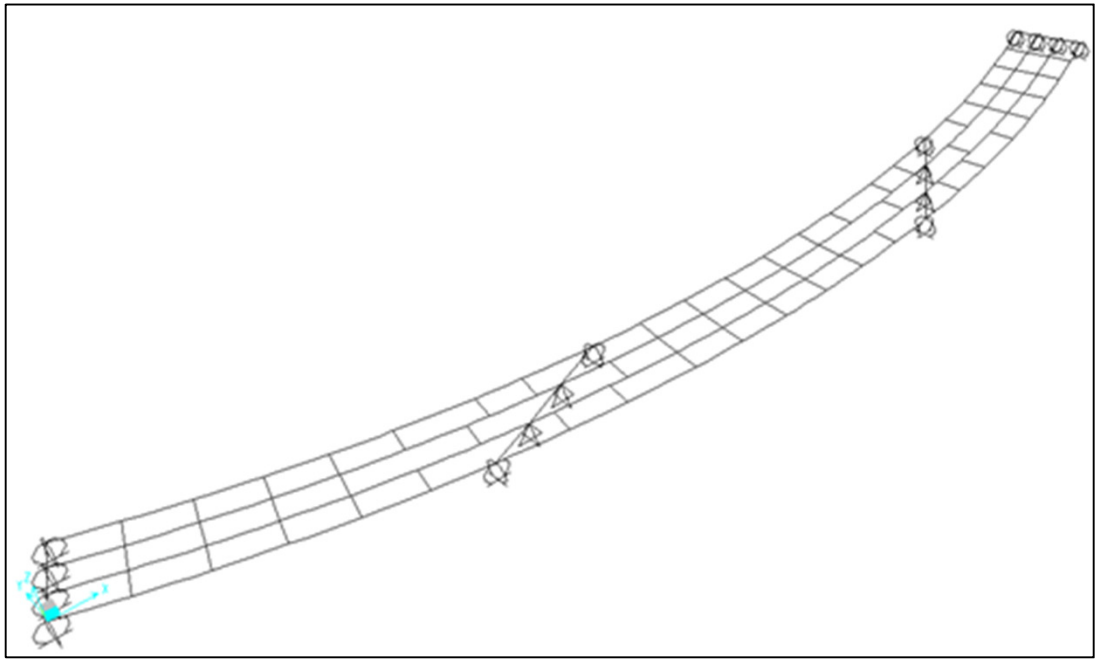
Approximate Methods

"Refined" Methods

Introduction: 2D-Grid Models



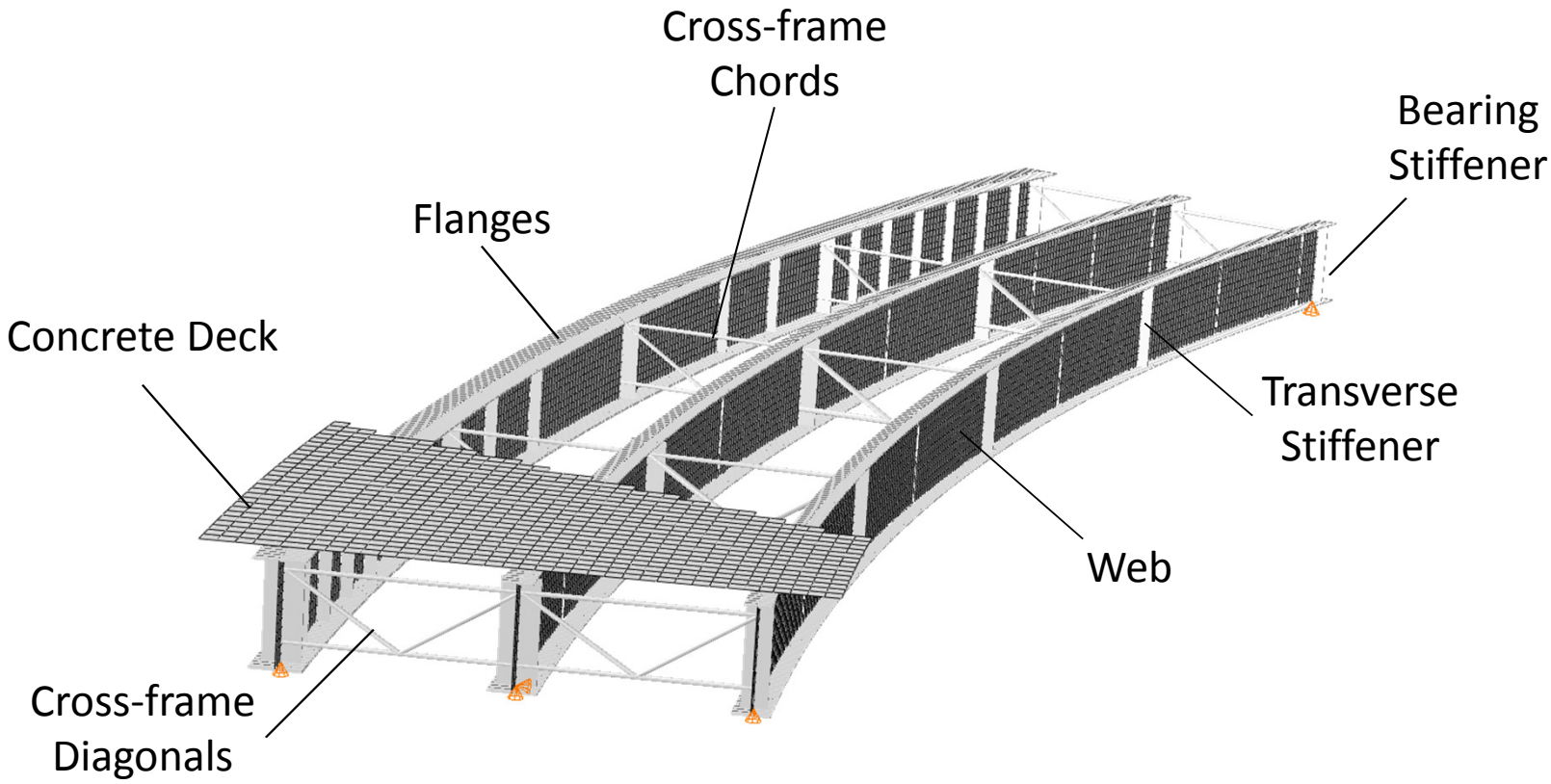
Typical element used in 2D-Grid Analysis



- In-plane representation of the bridge structure
- Girders and cross-frames are modeled with two-node (line) elements with 3 DOFs per node

Introduction: 3D FEA

Model Characteristics:



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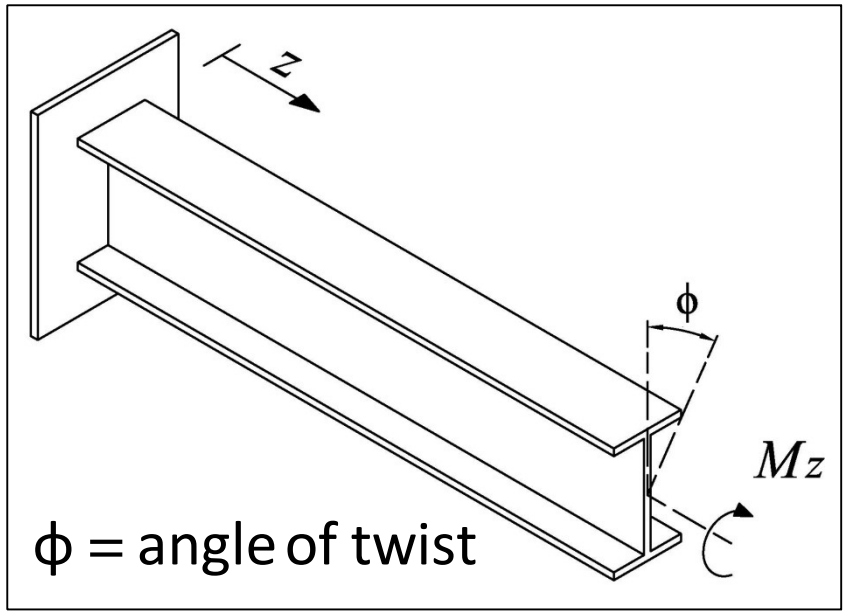
Current 2D Models: Torsional Stiffness

Governing Differential Equation for Torsion:



$$M_z(z) = M_s(z) + M_w(z)$$

$$M_z(z) = \boxed{GJ\frac{d\phi}{dz}} - \boxed{EC_w\frac{d^3\phi(z)}{dz^3}}$$

Warping contribution

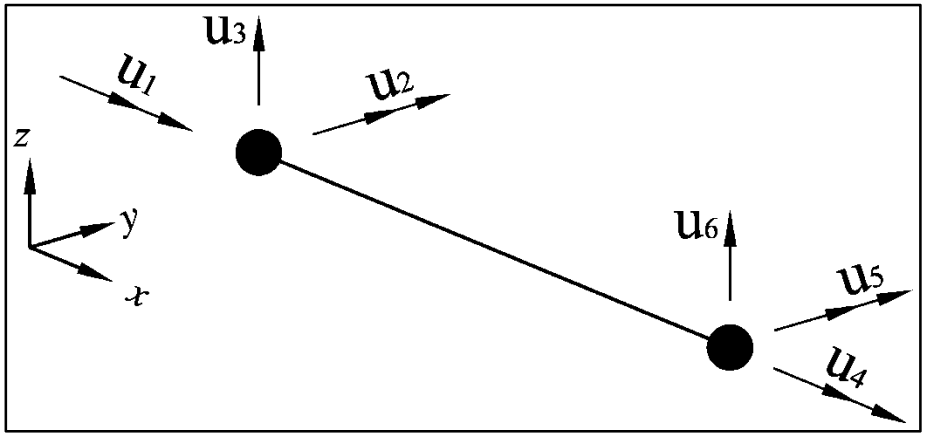
St. Venant (pure) torsion

$$M_z(z) = GJ\frac{d\phi}{dz}$$

$$M_z = \frac{GJ}{L}\phi$$ If torque, M_z, is constant

Current 2D Models: Torsional Stiffness

Stiffness Matrix Traditional 2D-Grid Element :



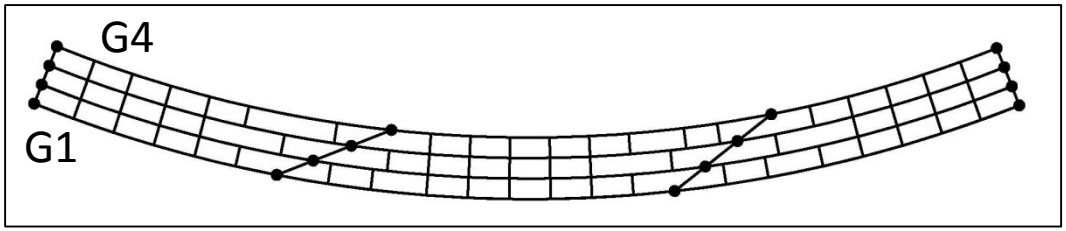
…but, in open-section thin walled members the major source of torsion resistance is flange warping

Curved and skewed bridges are subjected to torsion

$$k_{6\times6} = \begin{bmatrix} \boxed{\frac{GJ}{L}} & 0 & 0 & \boxed{-\frac{GJ}{L}} & 0 & 0 \\ 0 & \frac{4EI_x}{L} & -\frac{6EI_x}{L^2} & 0 & \frac{2EI_x}{L} & \frac{6EI_x}{L^2} \\ 0 & -\frac{6EI_x}{L^2} & \frac{12EI_x}{L^3} & 0 & -\frac{6EI_x}{L^2} & -\frac{12EI_x}{L^3} \\ \boxed{-\frac{GJ}{L}} & 0 & 0 & \boxed{\frac{GJ}{L}} & 0 & 0 \\ 0 & \frac{2EI_x}{L} & -\frac{6EI_x}{L^2} & 0 & \frac{4EI_x}{L} & \frac{6EI_x}{L^2} \\ 0 & \frac{6EI_x}{L^2} & -\frac{12EI_x}{L^3} & 0 & \frac{6EI_x}{L^2} & \frac{12EI_x}{L^3} \end{bmatrix}$$

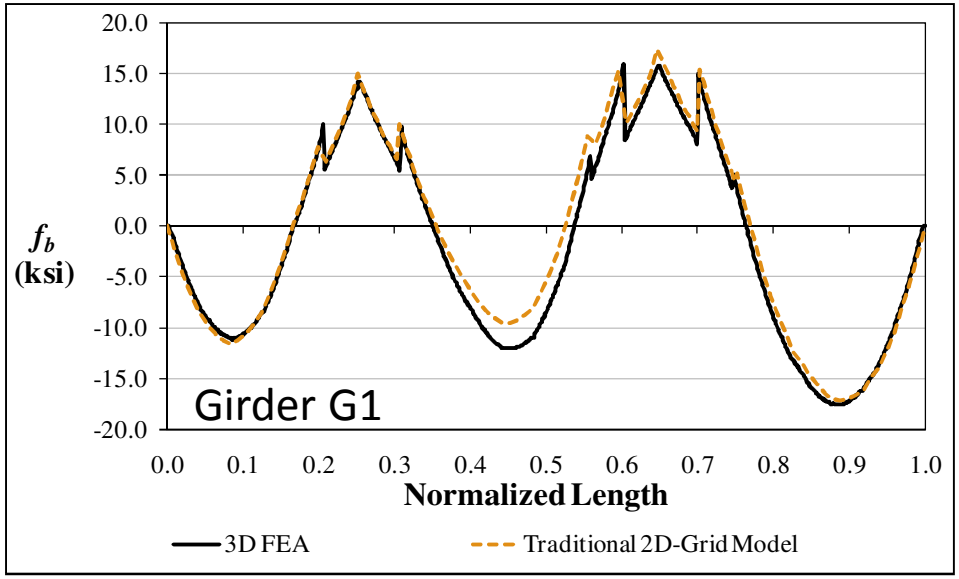
Current 2D Models: Torsional Stiffness

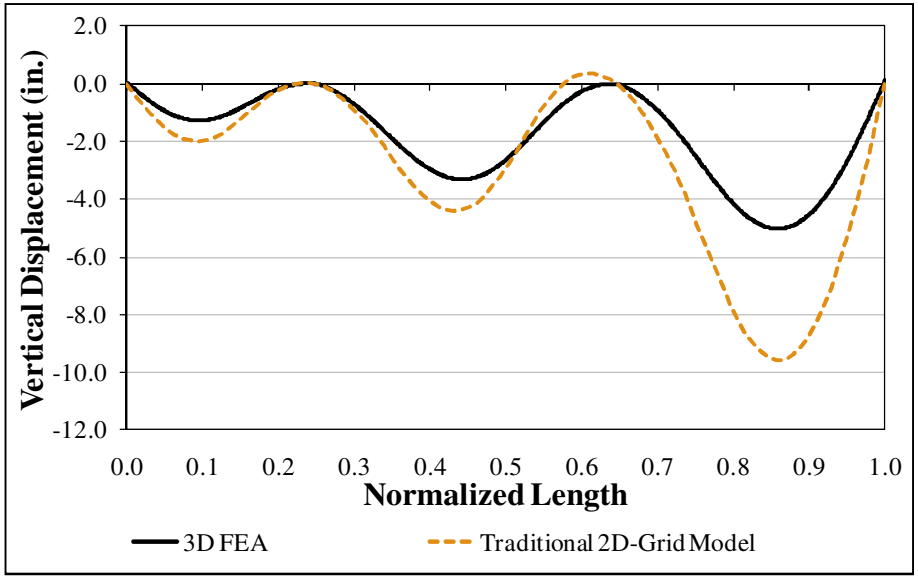
Case Study I:



Three-span, Curved & Skewed I-Girder Bridge

L_1 = 160 ft, L_2 = 210 ft, L_3 = 160 ft / R = 700 ft / w = 40.5 ft / θ_1= 0 ˚, θ_2= -60 ˚, θ_3= -60˚, θ_4= 0˚





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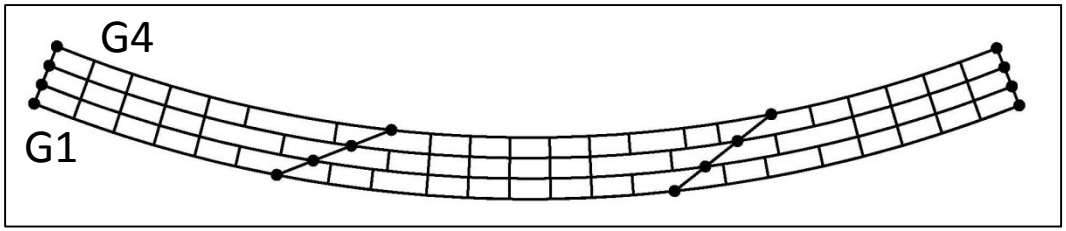

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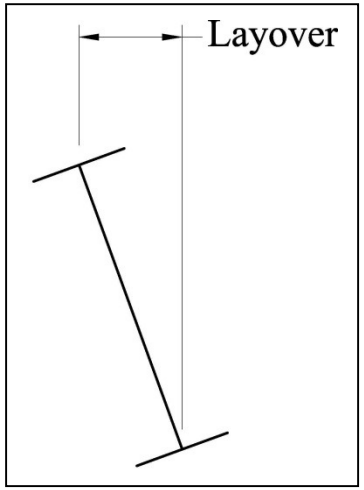
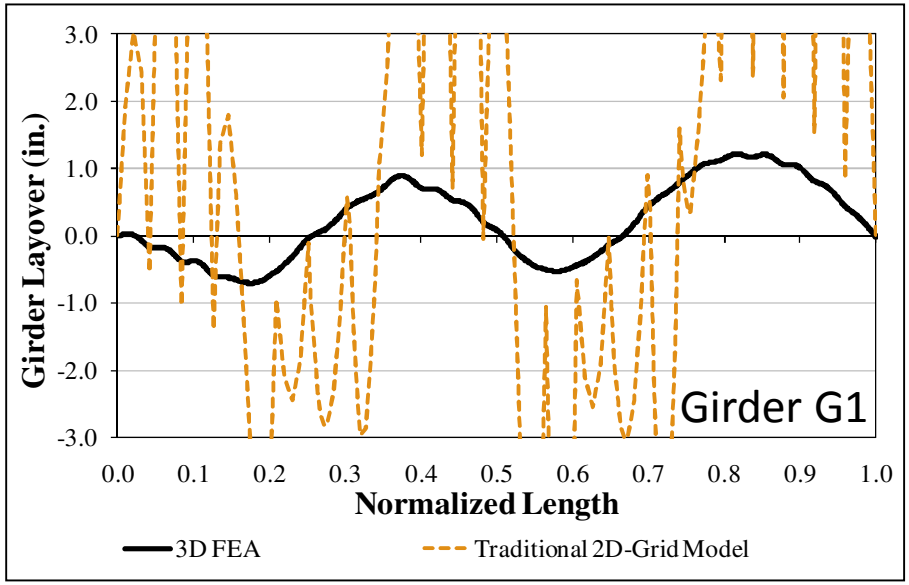
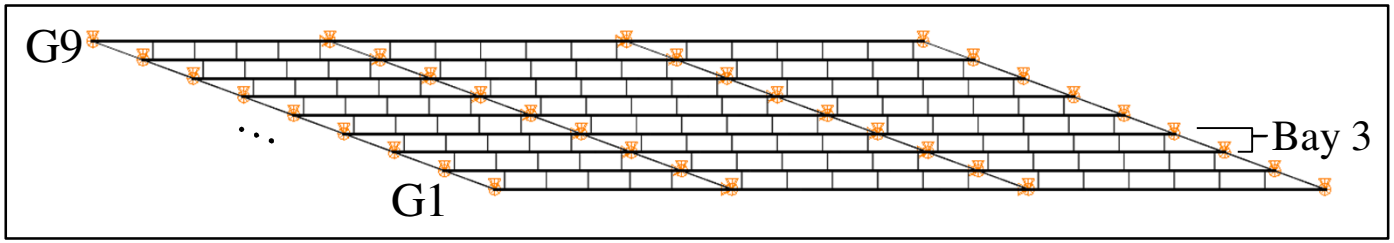




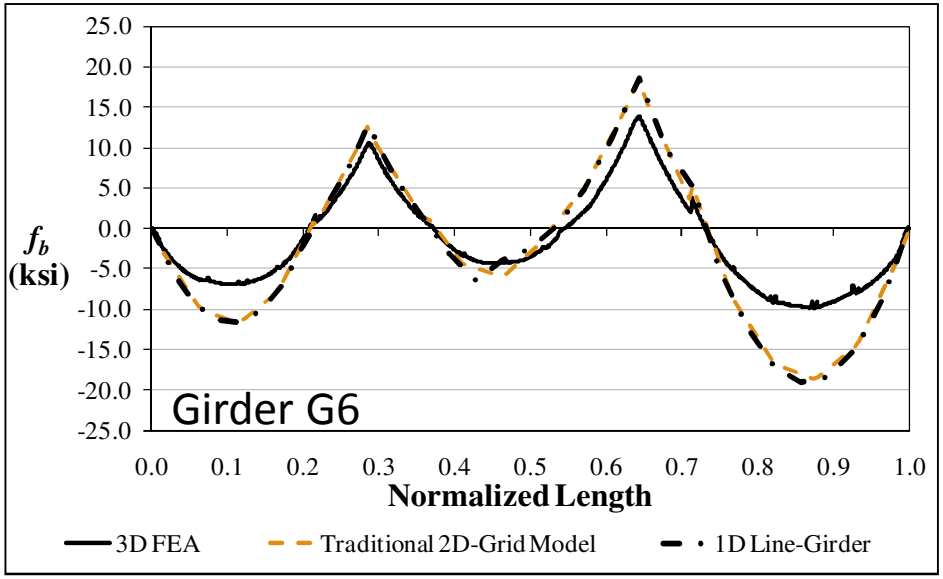
Current 2D Models: Torsional Stiffness

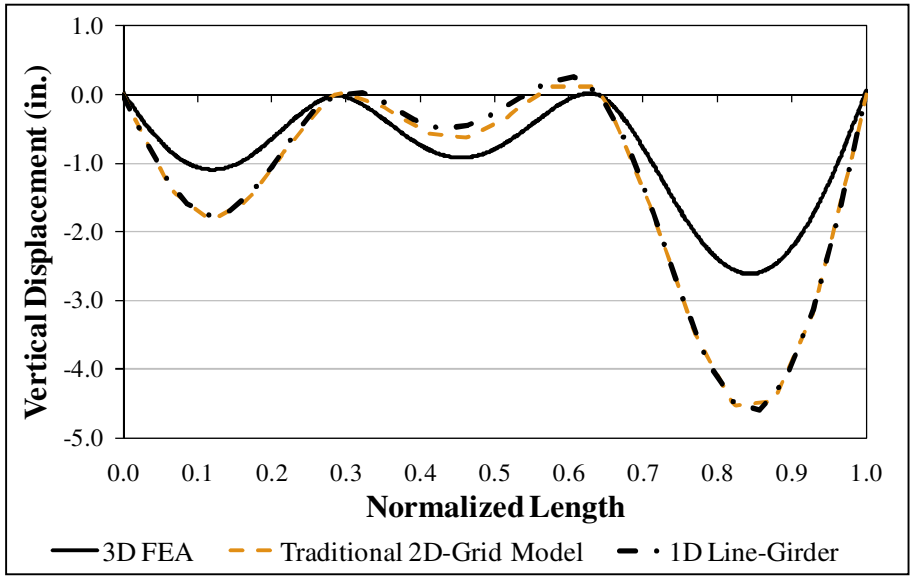
Case Study II:



Three-span, Straight & Skewed I-Girder Bridge

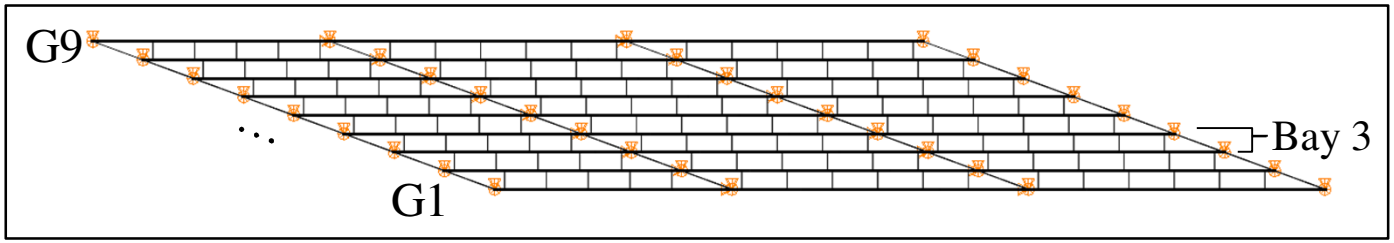
L_1 = 120 ft, L_2 = 150 ft, L_3 = 150 ft / w = 80 ft / θ_1 = 70 °, θ_2 = 70 °, θ_3 = 70 °, θ_4 = 70 °





Current 2D Models: Torsional Stiffness

Case Study II:



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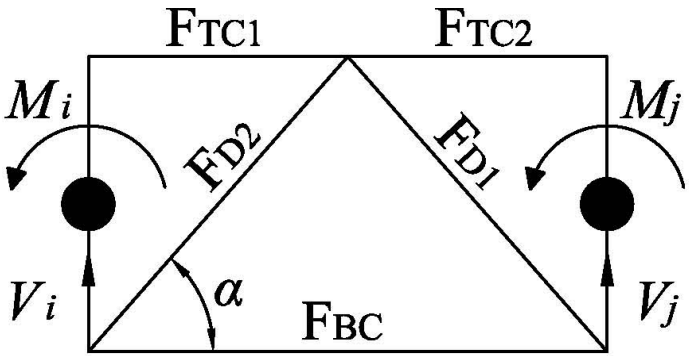
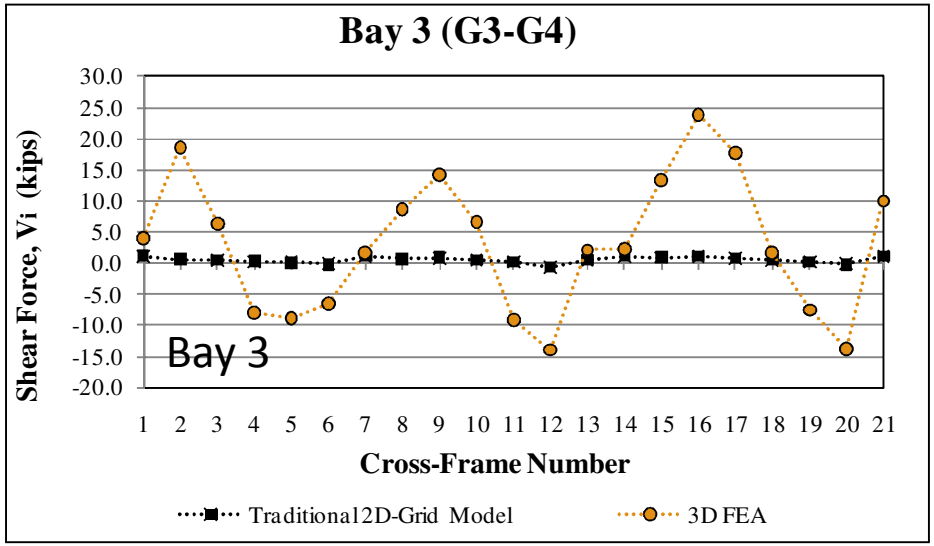
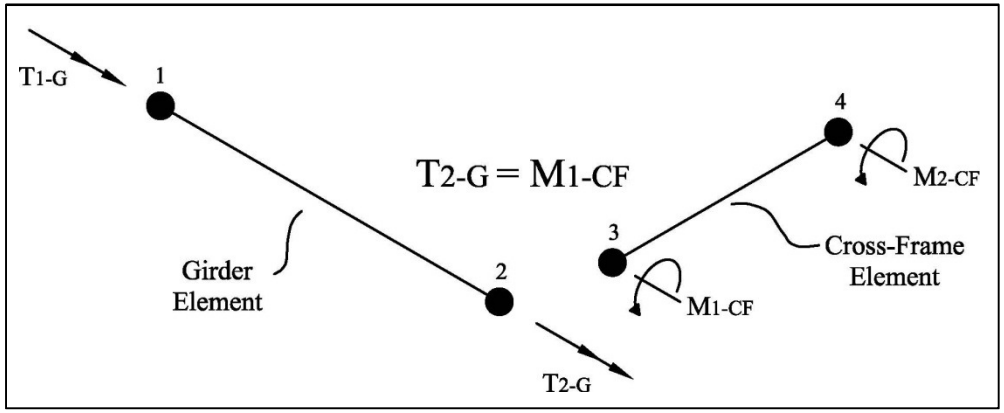


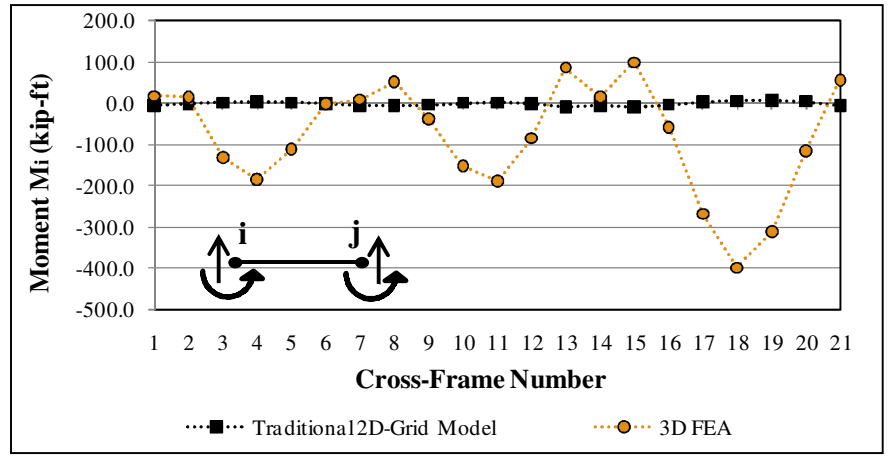
$$M_i = (F_{TC1} - F_{BC} - F_{D2} \cdot \sin\alpha) \cdot h/2$$
$$V_i = -F_{D2} \cdot \cos\alpha$$
$$M_j = (F_{BC} - F_{TC2} + F_{D1} \cdot \sin\alpha) \cdot h/2$$
$$V_j = -F_{D1} \cdot \cos\alpha$$

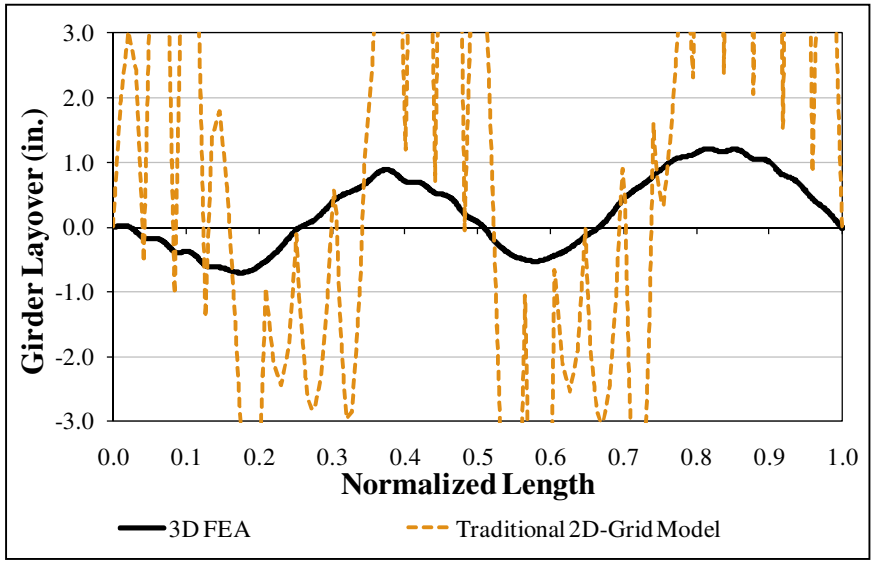


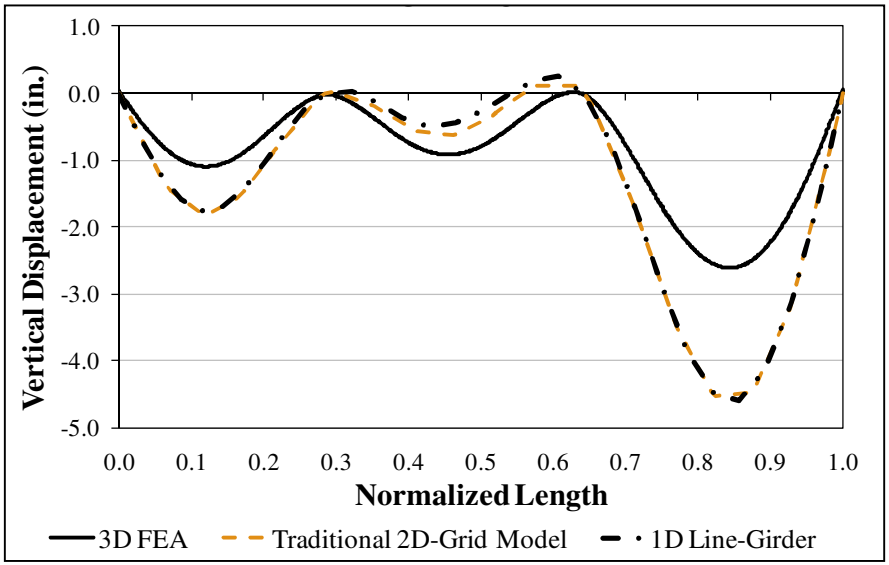
Current 2D Models: Torsional Stiffness

Effects of Torsion Model on the Structural Response Prediction:



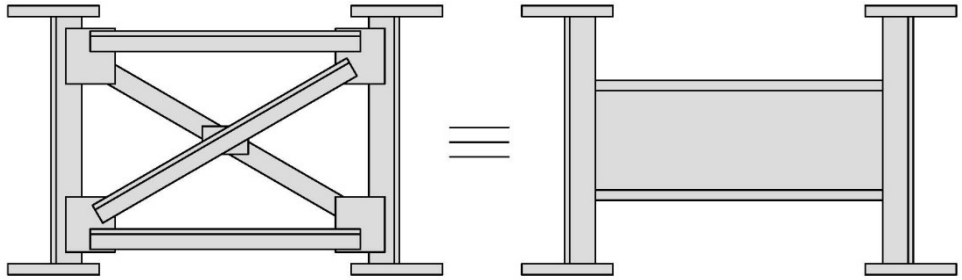






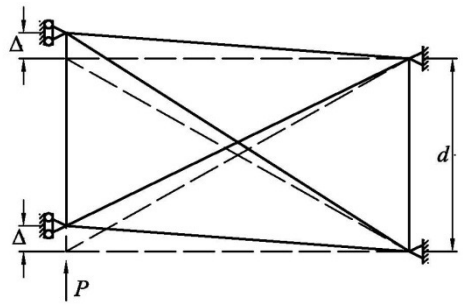
Current 2D Models: Cross-Frames

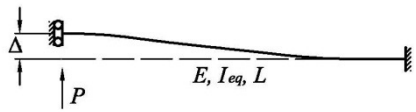
Determination of the Equivalent Beam Section Properties:



Cross-frame Euler-Bernoulli Beam

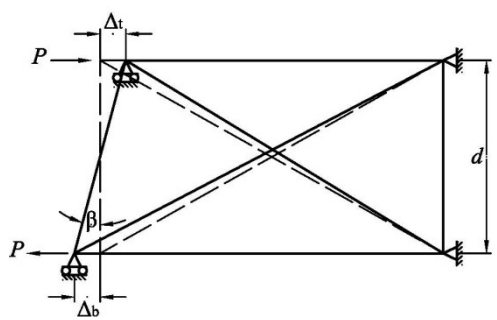
Shear Stiffness Model

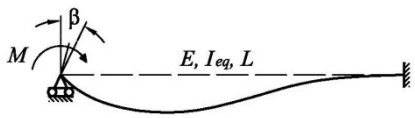




$$I_{eq} = \frac{PL^3}{12E\Delta}$$

Flexure Stiffness Model

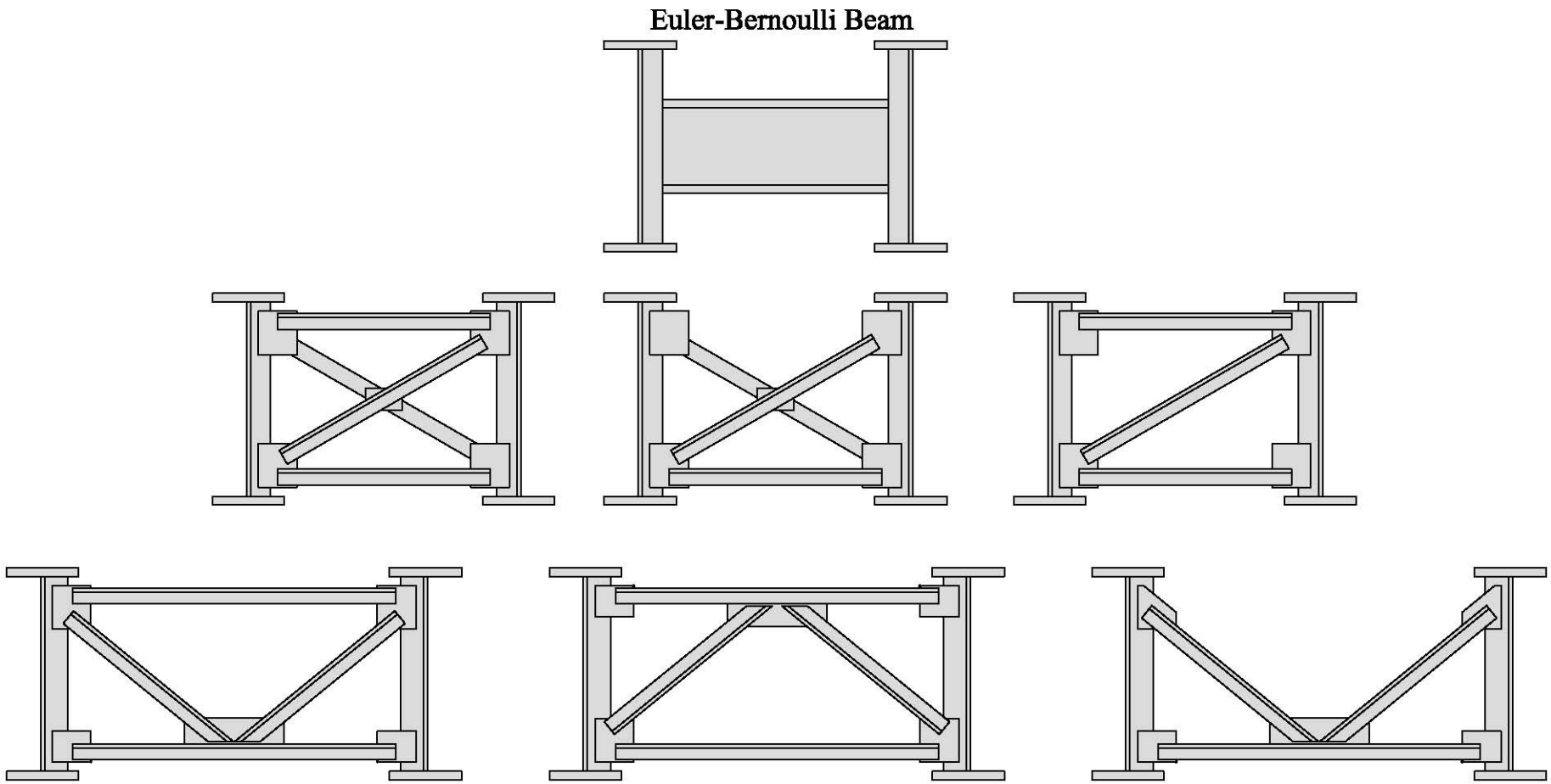




$$I_{eq} = \frac{ML}{4E\phi}$$

Current 2D Models: Cross-Frames

Determination of the Equivalent Beam Section Properties:



Current 2D Models: Limitations

Summary:

- With current practices, the effects of horizontal curvature and support skew may not be captured properly
- The limitations of the **torsional girder stiffness model** and **cross-frame beam model** can result in inaccurate predictions of:
    - Cross-frame forces
    - Flange lateral bending stresses, f_ℓ
    - Girder layovers
    - Major-axis bending stresses, f_b
    - Vertical displacements

Is it possible to overcome these two limitations?

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Improved 2D Models: Torsional Stiffness

Equivalent Torsion Constant, J_{eq}:

$$T = \boxed{GJ\frac{d\emptyset}{dz}} - \boxed{E\frac{I_f h^2}{2}\frac{d^3\emptyset}{dz^3}}$$

Warping contribution (Neglected in 2D models)

St. Venant (pure) torsion

$$J_{eq} = J\left\{1 - \frac{\sinh(pL)}{pL} + \frac{[\cosh(pL) \quad -1]^2}{pL\sinh(pL)}\right\}$$

Warping Fixed at both Ends

$$J_{eq} = J\left\{1 - \frac{\sinh(pL)}{pL\cosh(pL)}\right\}^{-1}$$

Warping Fixed at One End and Free at the Other End

Ahmed & Weisberger (1996)

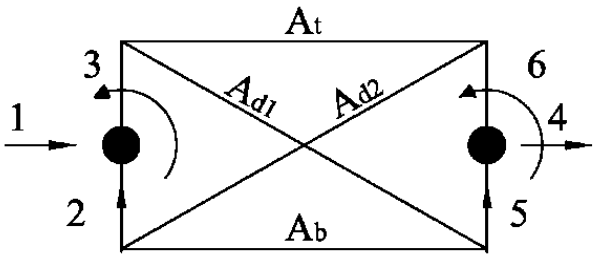
$$p = \sqrt{\frac{GJ}{EC_w}}$$

$$T = GJ_{eq}\frac{d\emptyset}{dz}$$

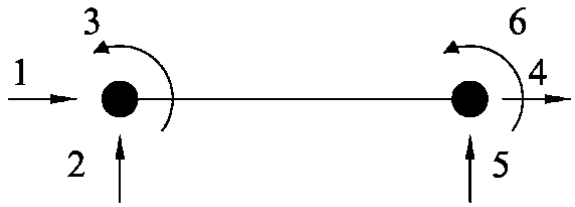
Improved 2D Models: Cross-Frames

Development of Two-Node Cross-Frame Elements:

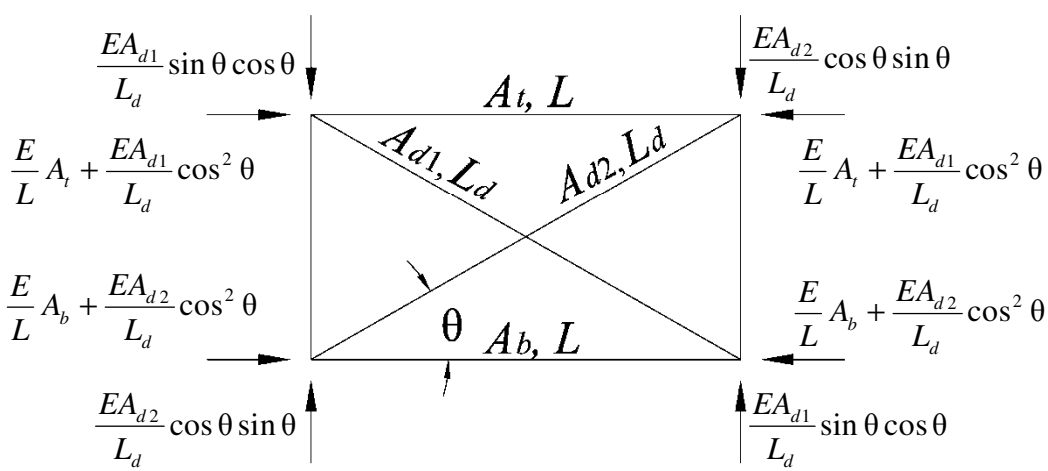
Physical Cross-Frame



Two-Node Element



Cross-Frame Coefficients – DOF 1:



$$k_{11}:\quad \frac{E}{L}\left(A_b + A_t\right) + \frac{EL^2}{L_d^{\,3}}\left(A_{d1} + A_{d2}\right)$$

$$k_{21}:\quad \frac{ELh}{L_d^{\,3}}\left(A_{d2} - A_{d1}\right)$$

$$k_{31}:\quad \frac{Eh}{2L}\left(A_b - A_t\right) + \frac{EhL^2}{2L_d^{\,3}}\left(A_{d2} - A_{d1}\right)$$

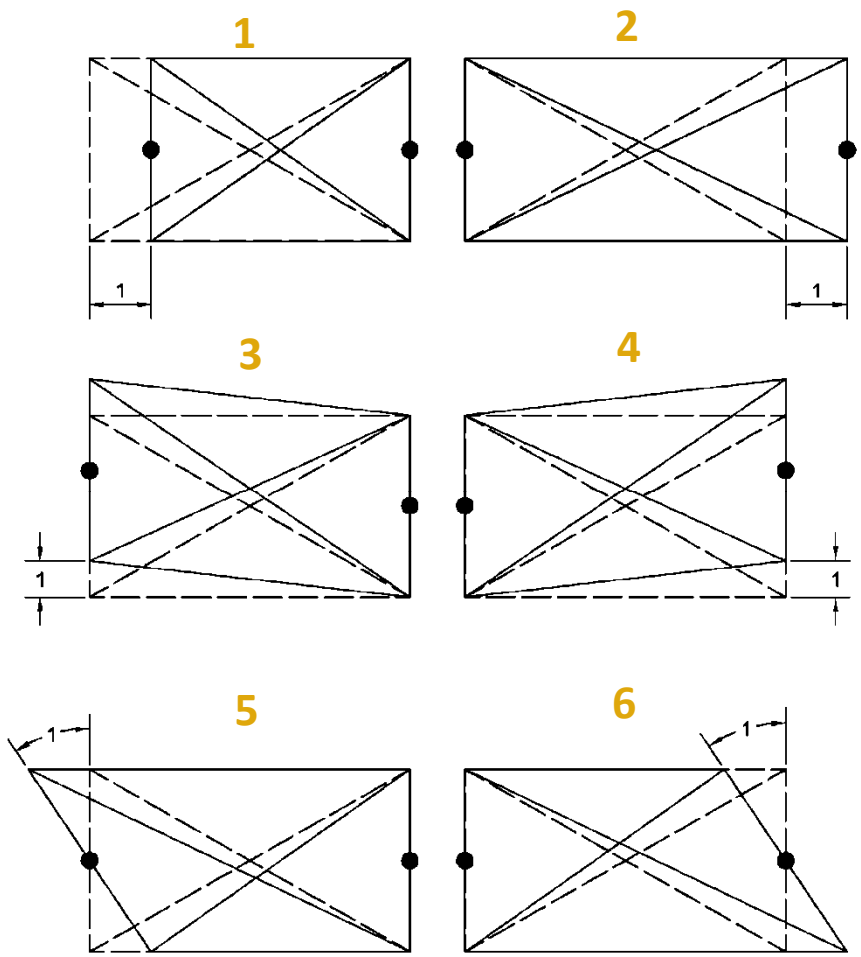
$$k_{41}:\quad -\frac{E}{L}\left(A_b + A_t\right) - \frac{EL^2}{L_d^{\,3}}\left(A_{d1} + A_{d2}\right)$$

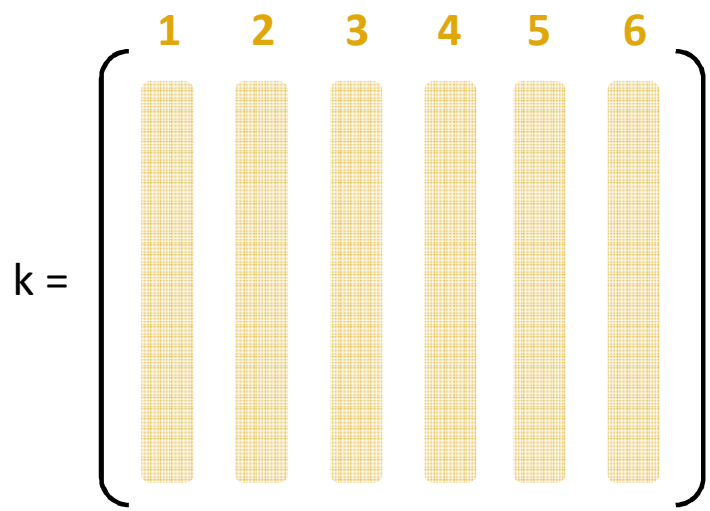
$$k_{51}:\quad \frac{ELh}{L_d^{\,3}}\left(A_{d1} - A_{d2}\right)$$

$$k_{61}:\quad \frac{Eh}{2L}\left(A_t - A_b\right) + \frac{EhL^2}{2L_d^{\,3}}\left(A_{d2} - A_{d1}\right)$$

Improved 2D Models: Cross-Frames

Development of Two-Node Cross-Frame Elements:





Improved 2D Models: Cross-Frames

Comparison Euler-Bernoulli Beam vs. X-Type Cross-Frame Element:

$$
k=E\begin{bmatrix}
\frac{1}{L}(A_b+A_t)+\frac{L^2}{L_d^3}(A_{d1}+A_{d2}) & \frac{Lh}{L_d^3}(A_{d2}-A_{d1}) & \frac{h}{2L}(A_b-A_t)+\frac{hL^2}{2L_d^3}(A_{d2}-A_{d1}) & -\frac{1}{L}(A_b+A_t)-\frac{L^2}{L_d^3}(A_{d1}+A_{d2}) & \frac{Lh}{L_d^3}(A_{d1}-A_{d2}) & \frac{h}{2L}(A_t-A_b)+\frac{hL^2}{2L_d^3}(A_{d2}-A_{d1}) \\
\frac{Lh}{L_d^3}(A_{d2}-A_{d1}) & \frac{h^2}{L_d^3}(A_{d1}+A_{d2}) & \boxed{\frac{h^2L}{2L_d^3}(A_{d1}+A_{d2})} & \frac{Lh}{L_d^3}(A_{d1}-A_{d2}) & -\frac{h^2}{L_d^3}(A_{d1}+A_{d2}) & \frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) \\
\frac{h}{2L}(A_b-A_t)+\frac{hL^2}{2L_d^3}(A_{d2}-A_{d1}) & \frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) & \frac{h^2}{4L}(A_b+A_t)+\frac{h^2L^2}{4L_d^3}(A_{d1}+A_{d2}) & \frac{h}{2L}(A_t-A_b)+\frac{hL^2}{2L_d^3}(A_{d1}-A_{d2}) & -\frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) & -\frac{h^2}{4L}(A_b+A_t)+\frac{h^2L^2}{4L_d^3}(A_{d1}+A_{d2}) \\
-\frac{1}{L}(A_b+A_t)-\frac{L^2}{L_d^3}(A_{d1}+A_{d2}) & \frac{Lh}{L_d^3}(A_{d1}-A_{d2}) & \frac{h}{2L}(A_t-A_b)+\frac{hL^2}{2L_d^3}(A_{d1}-A_{d2}) & \frac{1}{L}(A_b+A_t)+\frac{L^2}{L_d^3}(A_{d1}+A_{d2}) & \frac{Lh}{L_d^3}(A_{d2}-A_{d1}) & \frac{h}{2L}(A_b-A_t)+\frac{hL^2}{2L_d^3}(A_{d1}-A_{d2}) \\
\frac{Lh}{L_d^3}(A_{d1}-A_{d2}) & -\frac{h^2}{L_d^3}(A_{d1}+A_{d2}) & -\frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) & \frac{Lh}{L_d^3}(A_{d2}-A_{d1}) & \frac{h^2}{L_d^3}(A_{d1}+A_{d2}) & -\frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) \\
\frac{h}{2L}(A_t-A_b)+\frac{hL^2}{2L_d^3}(A_{d2}-A_{d1}) & \frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) & -\frac{h^2}{4L}(A_b+A_t)+\frac{h^2L^2}{4L_d^3}(A_{d1}+A_{d2}) & \frac{h}{2L}(A_b-A_t)+\frac{hL^2}{2L_d^3}(A_{d1}-A_{d2}) & -\frac{h^2L}{2L_d^3}(A_{d1}+A_{d2}) & \frac{h^2}{4L}(A_b+A_t)+\frac{h^2L^2}{4L_d^3}(A_{d1}+A_{d2})
\end{bmatrix}
$$

$$
k=E\begin{bmatrix}
\frac{A_{eq}}{L} & \boxed{0} & \boxed{0} & -\frac{A_{eq}}{L} & \boxed{0} & \boxed{0} \\
\boxed{0} & \frac{12I_{eq}}{L^3} & \frac{6I_{eq}}{L^2} & \boxed{0} & -\frac{12I_{eq}}{L^3} & \frac{6I_{eq}}{L^2} \\
\boxed{0} & \frac{6I_{eq}}{L^2} & \frac{4I_{eq}}{L} & \boxed{0} & -\frac{6I_{eq}}{L^2} & \frac{2I_{eq}}{L} \\
-\frac{A_{eq}}{L} & \boxed{0} & \boxed{0} & \frac{A_{eq}}{L} & \boxed{0} & \boxed{0} \\
\boxed{0} & -\frac{12I_{eq}}{L^3} & -\frac{6I_{eq}}{L^2} & \boxed{0} & \frac{12I_{eq}}{L^3} & -\frac{6I_{eq}}{L^2} \\
\boxed{0} & \frac{6I_{eq}}{L^2} & \frac{2I_{eq}}{L} & \boxed{0} & -\frac{6I_{eq}}{L^2} & \frac{4I_{eq}}{L}
\end{bmatrix}
$$

k_{11}: $A_{eq}=(A_b+A_t)+\left(L^3/L_d^3\right)(A_{d1}+A_{d2})$

Eight independent terms

k_{12}: $0=\left(Lh/L_d^3\right)(A_{d2}-A_{d1})$

k_{13}: $0=(h/2L)(A_b-A_t)+\left(hL^2/2L_d^3\right)(A_{d2}-A_{d1})$

k_{16}: $0=(h/2L)(A_t-A_b)+\left(hL^2/2L_d^3\right)(A_{d2}-A_{d1})$

k_{22}: $I_{eq}=\left(h^2L^3/12L_d^3\right)(A_{d1}+A_{d2})$ —— Shear stiffness method

k_{23}: $I_{eq}=\left(h^2L^3/12L_d^3\right)(A_{d1}+A_{d2})$

k_{33}: $I_{eq}=\left(h^2/16\right)(A_b+A_t)+\left(h^2L^3/16L_d^3\right)(A_{d1}+A_{d2})$ —— Flexure stiffness method

k_{36}: $I_{eq}=\left(-h^2/8\right)(A_b+A_t)+\left(h^2L^3/8L_d^3\right)(A_{d1}+A_{d2})$

Improved 2D Models: Cross-Frames

Finite Element Editor...

Profile | Required Input | Stiffness Matrix | Self Weight | Edit Stiffness Matrix

		Node 1						Node 2					
		Tx	Ty	Tz	Rx	Ry	Rz	Tx	Ty	Tz	Rx	Ry	Rz
Node 1	Tx	`E*(At+Ab)/L+(E*L^2/Ld^3)*(Ad1+Ad2)`	`(E*L*h/Ld^3)*(Ad2-Ad1)`	`0`	`0`	`0`	`E*h*(Ab-At)/(2*L)+(E*h*L^2/(2*Ld^3))*(Ad2-Ad1)`	`-E*(At+Ab)/L-(E*L^2/Ld^3)*(Ad1+Ad2)`	`(E*L*h/Ld^3)*(Ad1-Ad2)`	`0`	`0`	`0`	`E*h*(At-Ab)/(2*L)+(E*h*L^2/(2*Ld^3))*(Ad2-Ad1)`
	Ty		`(E*h^2/Ld^3)*(Ad2+Ad1)`	`0`	`0`	`0`	`(E*h^2*L/(2*Ld^3))*(Ad1+Ad2)`	`(E*h*L/Ld^3)*(Ad1-Ad2)`	`-(E*h^2/Ld^3)*(Ad2+Ad1)`	`0`	`0`	`0`	`(E*h^2*L/(2*Ld^3))*(Ad2+Ad1)`
	Tz			`12*E*(Ib+It)/L^3`	`6*E*h*(It-Ib)/L^3`	`-6*E*(Ib+It)/L^2`	`0`	`0`	`0`	`-12*E*(Ib+It)/L^3`	`6*E*h*(Ib-It)/L^3`	`-6*E*(Ib+It)/L^2`	`0`
	Rx				`(3*E*h^2/L^3)*(Ib+It)`	`(3*E*h/L^2)*(Ib-It)`	`0`	`0`	`0`	`6*E*h*(Ib-It)/L^3`	`-3*E*h^2*(Ib+It)/L^3`	`3*E*h*(Ib-It)/L^2`	`0`
	Ry					`(4*E/L)*(Ib+It)`	`0`	`0`	`0`	`(6*E/L^2)*(Ib+It)`	`(3*E*h/L^2)*(It-Ib)`	`(2*E/L)*(Ib+It)`	`0`
	Rz						`E*h^2*(At+Ab)/(4*L)+(E*h^2*L^2/(4*Ld^3))*(Ad1+Ad2)`	`E*h*(At-Ab)/(2*L)+(E*L^2*h/(2*Ld^3))*(Ad1-Ad2)`	`-(E*h^2*L/(2*Ld^3))*(Ad1+Ad2)`	`0`	`0`	`0`	`-E*h^2*(At+Ab)/(4*L)+(E*h^2*L^2/(4*Ld^3))*(Ad1+Ad2)`
Node 2	Tx							`E*(At+Ab)/L+(E*L^2/Ld^3)*(Ad1+Ad2)`	`(E*h*L/Ld^3)*(Ad2-Ad1)`	`0`	`0`	`0`	`E*h*(Ab-At)/(2*L)+(E*L^2*h/(2*Ld^3))*(Ad1-Ad2)`
	Ty								`(E*h^2/Ld^3)*(Ad1+Ad2)`	`0`	`0`	`0`	`-(E*h^2*L/(2*Ld^3))*(Ad1+Ad2)`
	Tz									`(12*E/L^3)*(Ib+It)`	`(6*E*h/L^3)*(It-Ib)`	`(6*E/L^2)*(Ib+It)`	`0`
	Rx										`(3*E*h^2/L^3)*(It+Ib)`	`(3*E*h/L^2)*(It-Ib)`	`0`
	Ry											`(4*E/L)*(Ib+It)`	`0`
	Rz												`E*h^2*(At+Ab)/(4*L)+(E*h^2*L^2/(4*Ld^3))*(Ad1+Ad2)`

`(E*h^2*L/(2*Ld^3))*(Ad1+Ad2)`

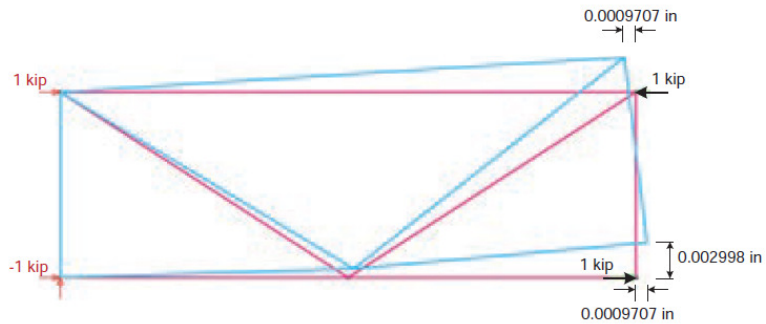
Improved 2D Models: Cross-Frames

Timoshenko Beam Approach

- **Alternate to custom finite elements**
- **Calculate equivalent moment of inertia, I_{eq}, from a pure bending model**
- **Calculate equivalent shear area, A_{seq} , from a pure shear model**
- **Implement I_{eq} and A_{seq} in a shear-deformable element (Timoshenko beam) in the 2D model**

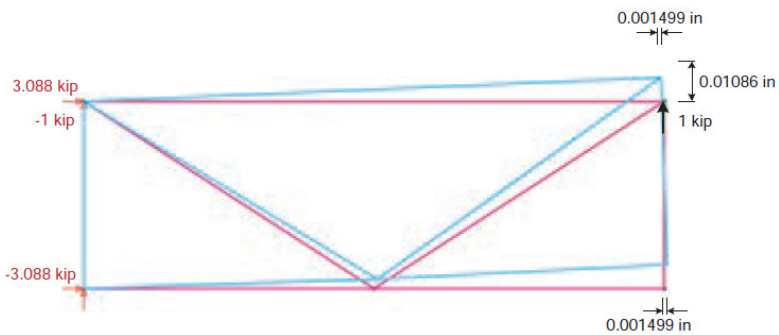
Improved 2D Models: Cross-Frames

Timoshenko Beam Approach



$$\theta = 2(0.0009707)/34 = 0.0000571 = ML/EI_{eq} = 34(105)/29000I_{eq}$$
$$I_{eq} = 2156 \text{ in}^4$$

Figure 3-17. Calculation of equivalent moment of inertia based on pure bending.



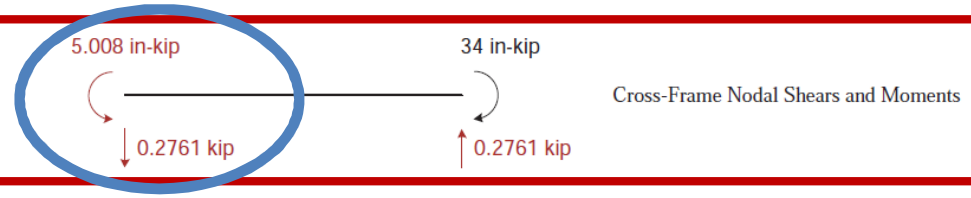
$$\Delta = 0.01086 \text{ in} = VL^3/3EI_{eq} + VL/GA_{eq}$$
$$= 1(105)^3/3(29000)(2156) + (1)(105)(2.6)/29000A_{seq}$$
$$A_{seq} = 2.008 \text{ in}^2$$

Figure 3-18. Calculation of equivalent shear area based on tip loading of the cross-frame supported as a cantilever.

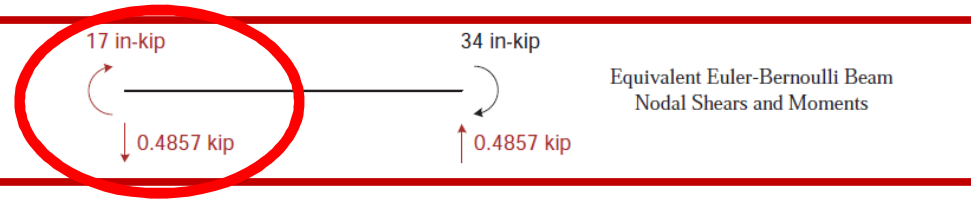
Improved 2D Models: Cross-Frames

Timoshenko Beam Approach

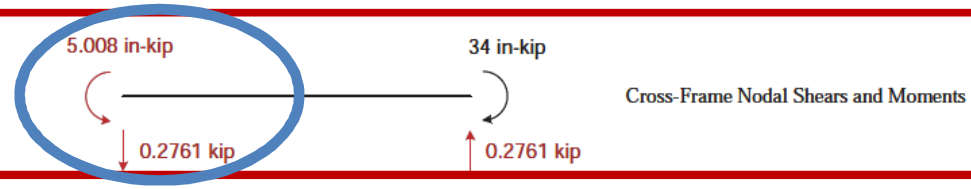
- 3D model



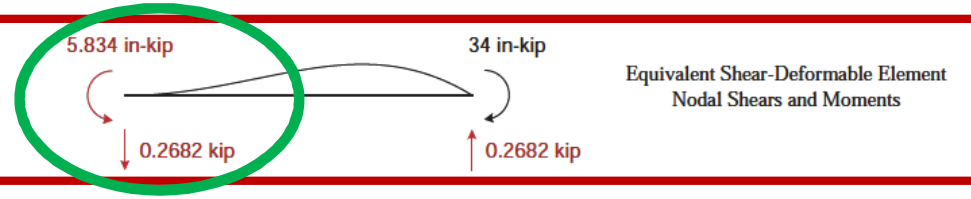
- Current 2D "Flexure Stiffness Method"



- 3D model



- Proposed Timoshenko Beam method



Presentation Outline

- Introduction
- Current 2D Models
- **Improved 2D Models**
- Case Studies
- Conclusions

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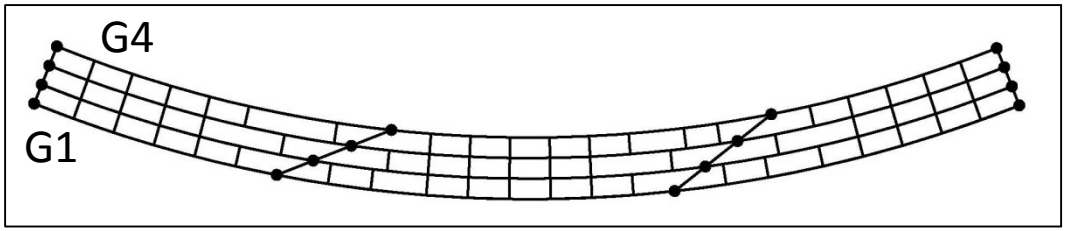
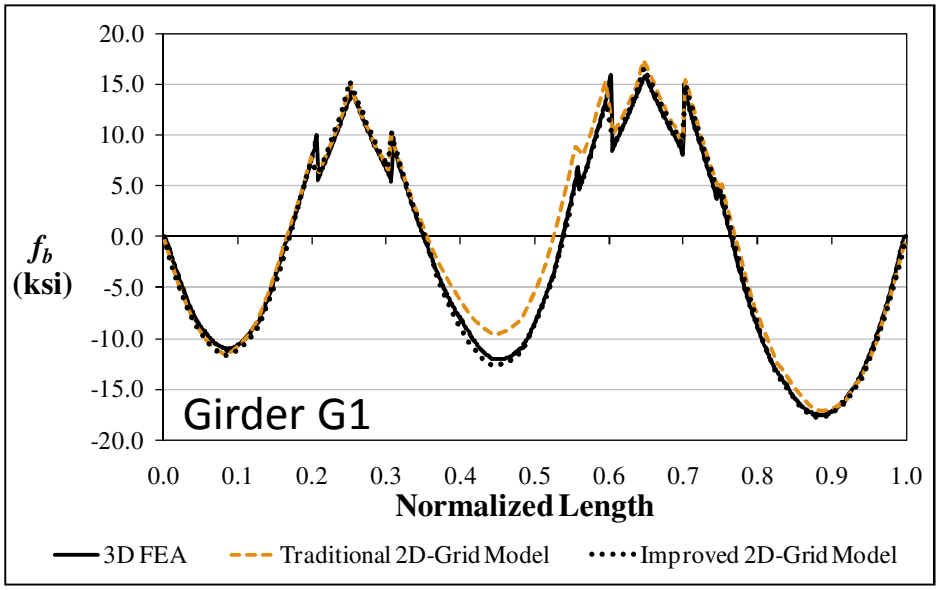
Case Studies

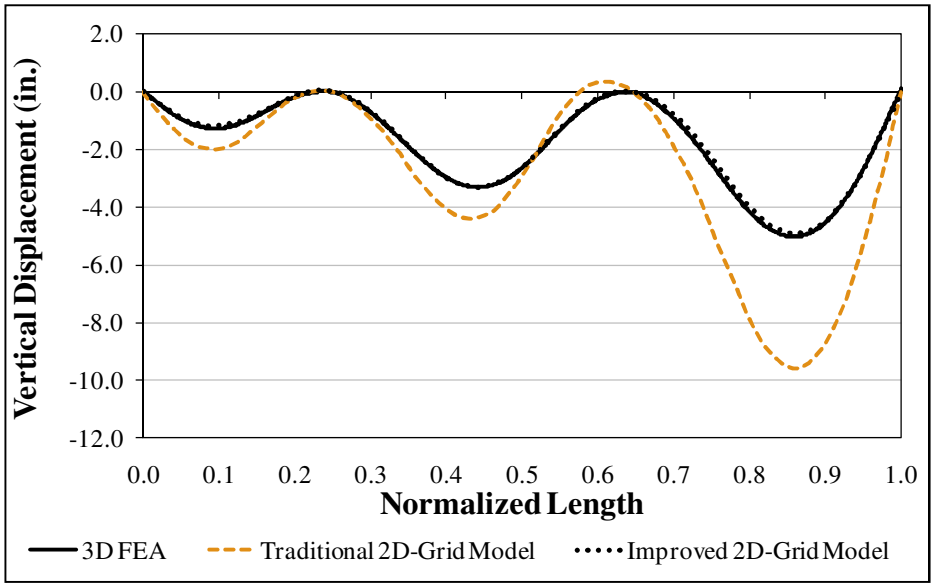
Case Study I:



Three-span, Curved & Skewed I-Girder Bridge

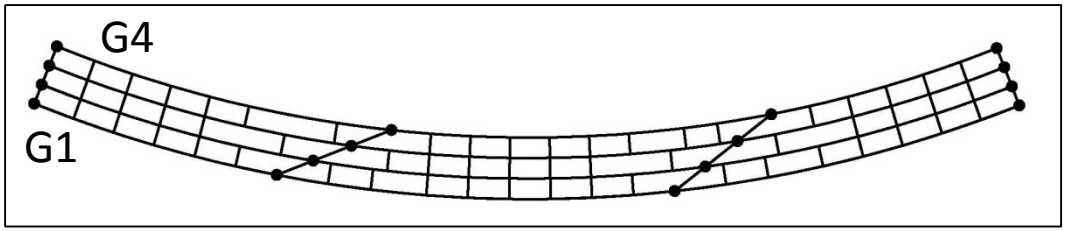
L_1 = 160 ft, L_2 = 210 ft, L_3 = 160 ft / R = 700 ft / w = 40.5 ft / θ_1= 0 ˚, θ_2= -60 ˚, θ_3= -60˚, θ_4= 0˚





Case Studies

Case Study I:



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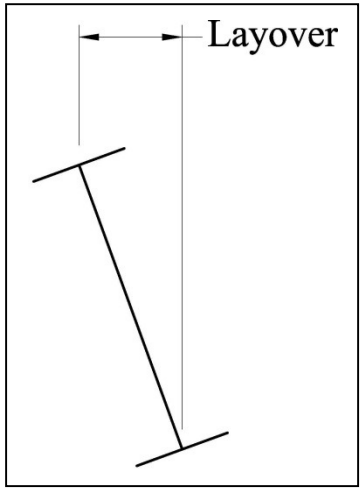
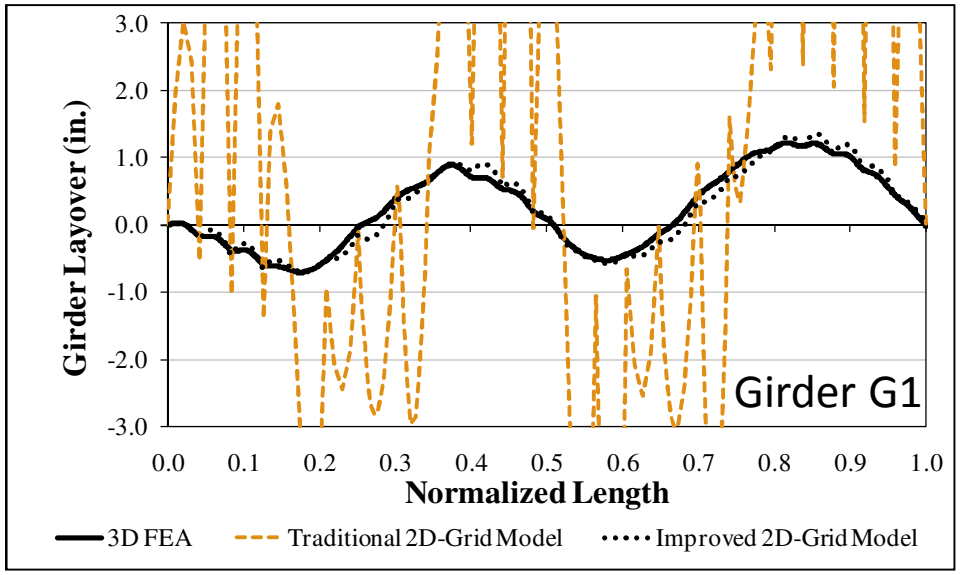
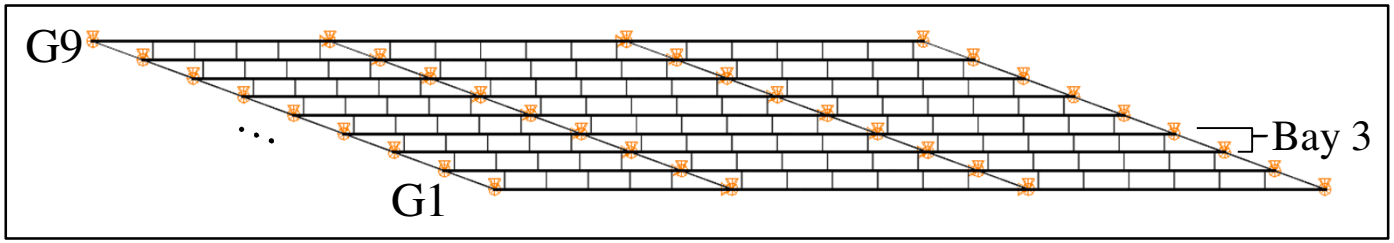




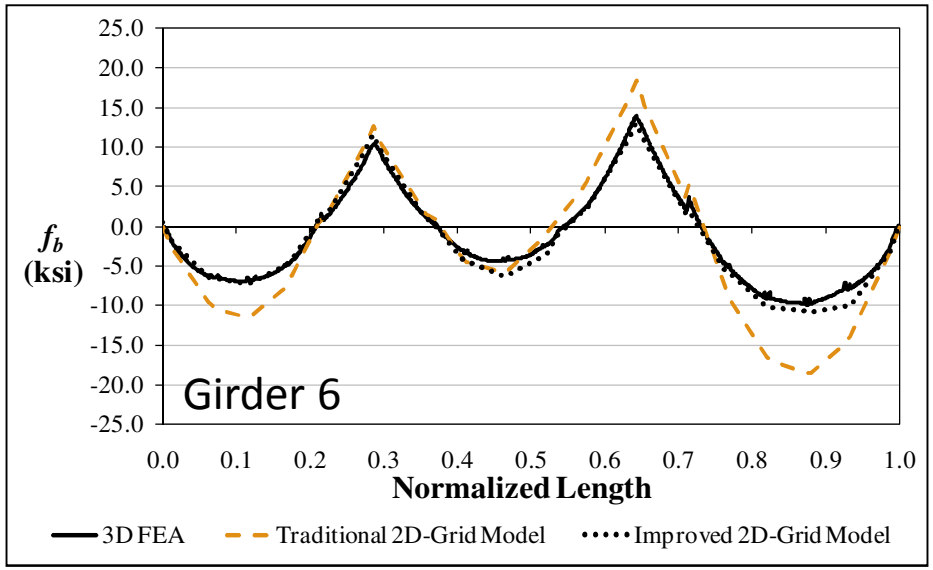
Case Studies

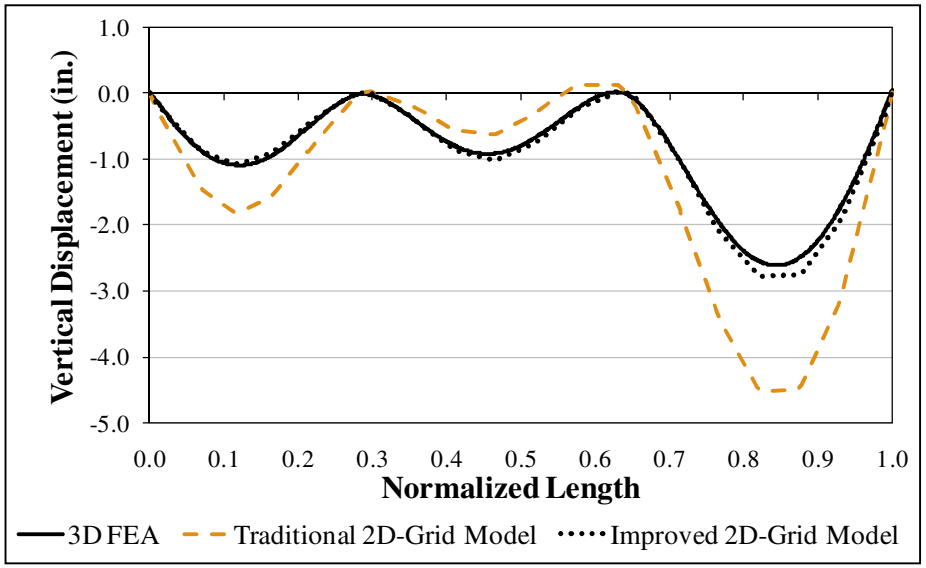
Case Study II:



Three-span, Straight & Skewed I-Girder Bridge

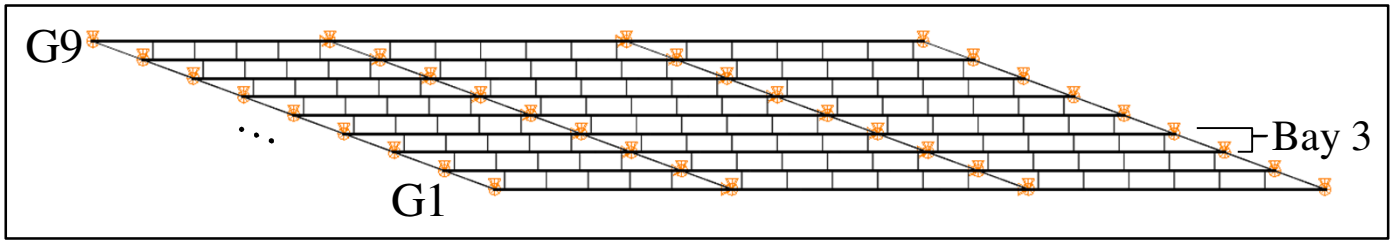
L_1 = 120 ft, L_2 = 150 ft, L_3 = 150 ft / w = 80 ft / θ_1 = 70 °, θ_2 = 70 °, θ_3 = 70 °, θ_4 = 70 °





Case Studies

Case Study II:



Three-span, Straight & Skewed I-Girder Bridge

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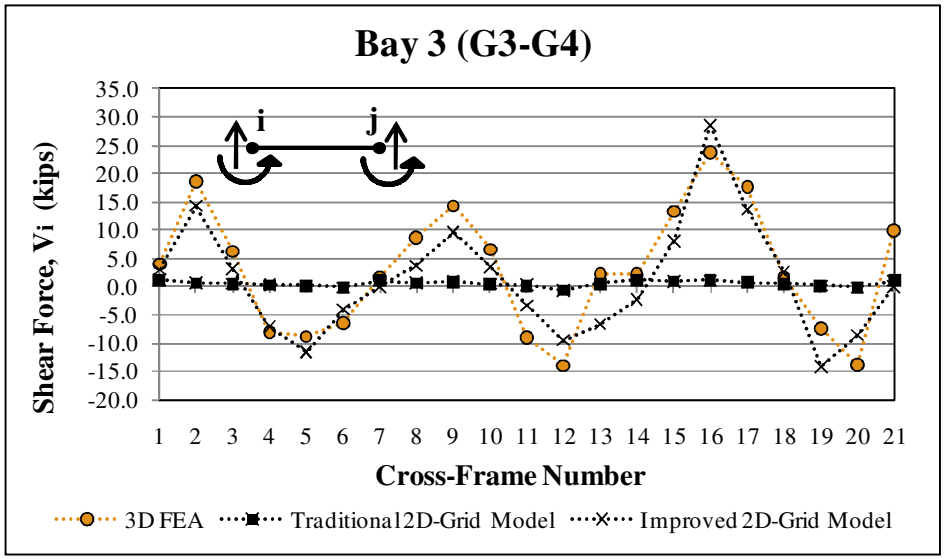

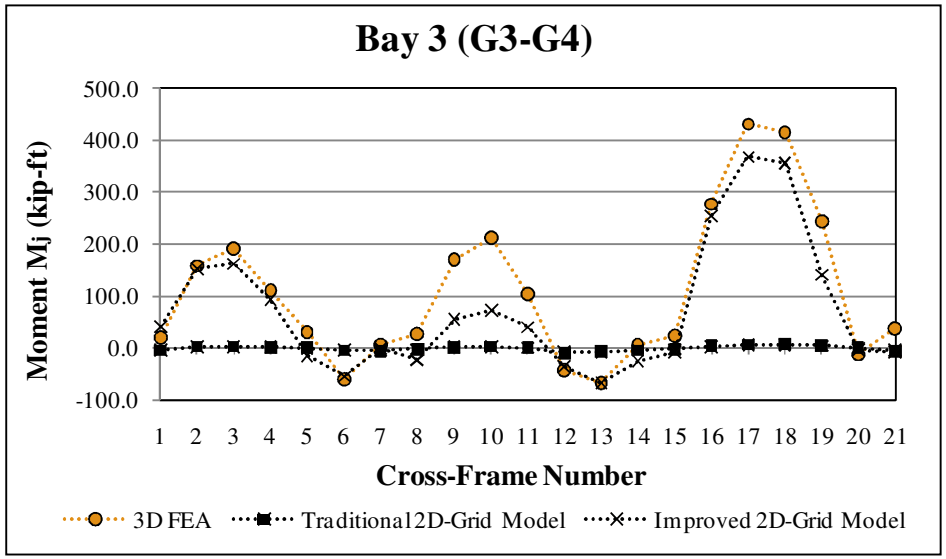

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Conclusions

- **Current 2D models are not accurate for all curved and skewed I-girder bridges during construction**
  - **Girder torsional stiffness not properly modeled**
  - **Cross-frame stiffness not properly modeled**
- **Improvements are possible**
  - **Can improve torsional stiffness modeling with J_{eq}**
  - **Can improve cross-frame stiffness modeling with**
- **Proposed improvements should be relatively easy for software providers to implement**

QUESTIONS?